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15MATDIP31

Third Semester B.E. Degree Examination, July/August 2022

Additional Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Express $\dfrac{(2-3i)(2+i)^2}{1+i}$ in the form of x + iy. **(06 Marks)**

b. If $x+\dfrac{1}{x}=2\cos\alpha$ then prove that $x^n+\dfrac{1}{x^n}=2\cos n\alpha$. **(05 Marks)**

c. Find the cosine of the angle between the vectors $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(05 Marks)**

OR

2 a. Find the Fourth roots of $1-i\sqrt{3}$ and represent them on an Argand plane. **(06 Marks)**

b. Show that the vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+4\hat{j}-\hat{k}$ are co-planar. **(05 Marks)**

c. Prove that $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}]=2[\vec{a},\ \vec{b},\ \vec{c}]$. **(05 Marks)**

Module-2

3 a. Obtain the n^{th} derivative of $e^{ax}\cos(bx+c)$. **(06 Marks)**

b. Show that the curves $r=a(1+\cos\theta)$ and $r=a(1-\cos\theta)$ are orthogonal. **(05 Marks)**

c. If $u=x(1-y)$, $v=xy$ find the Jacobians $J=\dfrac{\partial(u,v)}{\partial(x,y)}$ and $J'=\dfrac{\partial(x,y)}{\partial(u,v)}$. **(05 Marks)**

OR

4 a. If $y=a\cos(\log x)+b\sin(\log x)$, prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$. **(06 Marks)**

b. If $u=\sin^{-1}\left(\dfrac{x^3-y^3}{x-y}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=2\operatorname{Tan} u$. **(05 Marks)**

c. If $z=xy^2+x^2y$, where $x=at^2$, $y=2at$. Find $\dfrac{dz}{dt}$. **(05 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} x\sin^6 x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^1\int_0^1 \dfrac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

OR

6 a. Evaluate $\int_0^1 x^5(1-x^2)^{\frac{5}{2}}\, x.dx$. **(06 Marks)**

b. Evaluate $\int_0^{2a}\int_0^{\frac{x^2}{4a}} xy \; dy\, dx$. **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^y xyz \; dx\, dy\, dz$. **(05 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = 2t^2\,\hat{i} + (t^2-4t)\hat{j} + (3t-5)\hat{k}$. Find the components of velocity and acceleration at t = 2. **(06 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$. **(05 Marks)**

c. Find $\text{div}\,\vec{f}$ for $\vec{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(05 Marks)**

OR

8 a. Find the unit tangent vector to the curve $\vec{r} = t^2\,\hat{i} + 2t\,\hat{j} - t^3\,\hat{k}$ at $t = \pm 1$. **(06 Marks)**

b. Find the unit normal vector to the surface xy + yz + zx = c at the point (-1, 2, 3). **(05 Marks)**

c. Show that $\vec{f} = (z + \sin y)\,\hat{i} + (x\cos y - z)\,\hat{j} + (x-y)\,\hat{k}$ is irrotational. **(05 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \text{Sin}\left(\frac{y}{x}\right)$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \text{Sin}\, x$. **(05 Marks)**

c. Solve $(x^2 + y)\,dx + (y^3 + x)\,dy = 0$. **(05 Marks)**

OR

10 a. Solve $\frac{dy}{dx} = (4x + y + 1)^2$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{2}{x}y = \frac{3x^2+1}{x^2}$. **(05 Marks)**

c. Solve $[y(1+\frac{1}{x}) + \text{Cos}\, y]\,dx + (x + \log x - x\,\text{Sin}\, y)\,dy = 0$. **(05 Marks)**

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